

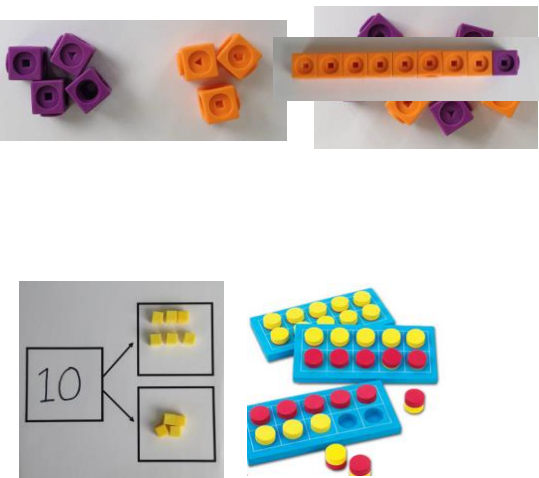
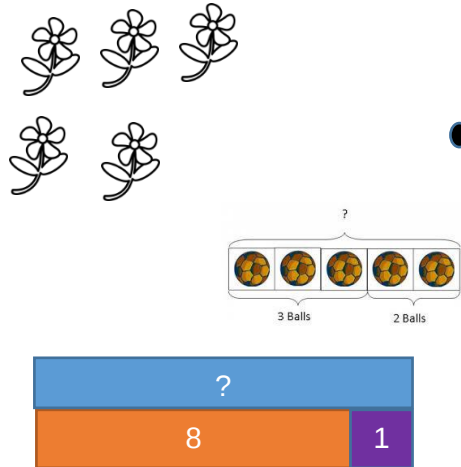
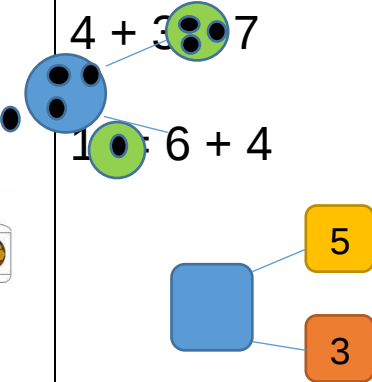
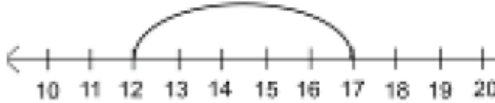


Key Images and Concrete Representations These are interchangeable and children should be regularly exposed to a range of these models to suit the learning.

Addition and subtraction	Part – part – whole cherry diagram Bar model Ten frames Counting on fingers Number lines (numbered and blank) Place value grid Dienes Place value counters Double sided counters
Multiplication and division	Counters Dienes Number lines Place value counters Using fingers

Addition

Children should be secure in number and place value, particularly partitioning number in different ways, before starting calculation strategies.

Objective and Strategies	Concrete	Pictorial	Abstract
Combining two parts to make a whole. Including being fluent in number bonds to 10.			
Starting at the bigger number and counting on		$12 + 5 = 17$  Start at the larger number on the number line and count on in ones or in	$5 + 12 = 17$ The strategy of using hands to count is appropriate! Place the larger number in your head and count on the smaller number to find your answer.

'Loop method' to support the addition of two 2-digit numbers

Compact method – no exchanging

42 + 34 = 76

40 + 2 + 30 + 4
70 + 6 = 76

Add together the ones first then add the tens.

24 + 15 = 39

100s 10s 1s

one jump to find the answer.

42 + 34 = 76

|||| : ||| ::
70 + 6 = 76

24 + 15 = 39

100s 10s 1s

Add the ones

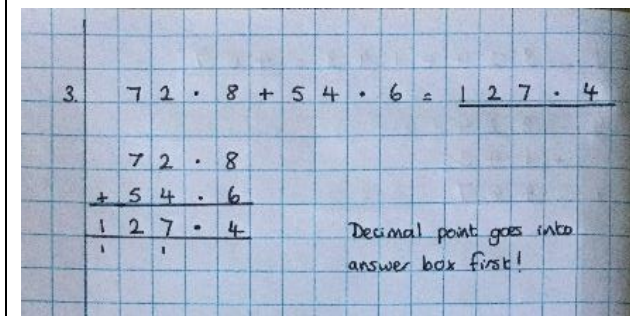
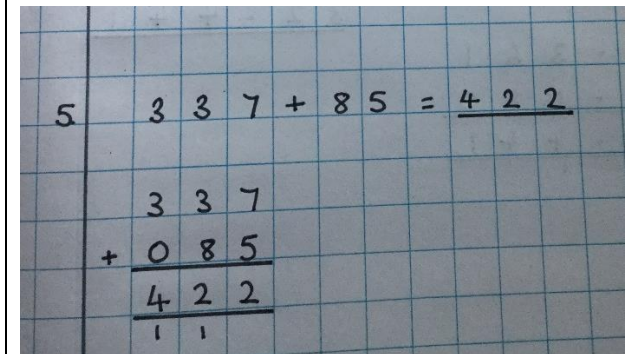
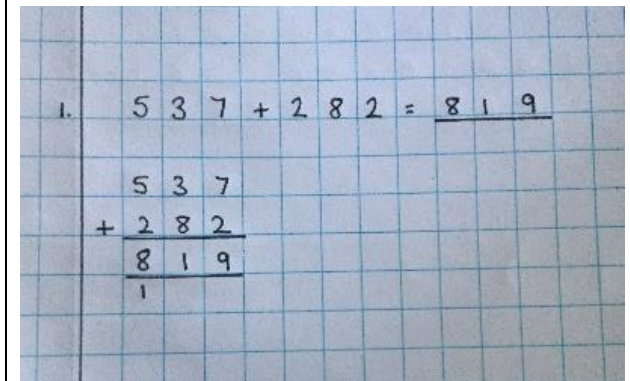
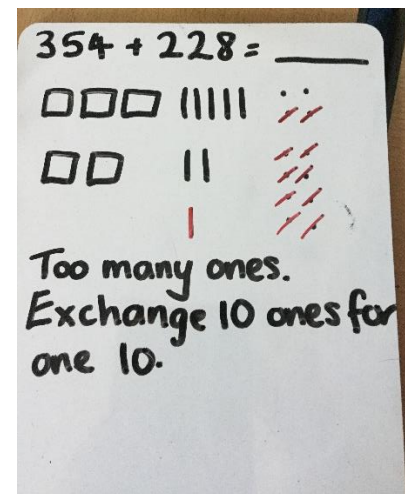
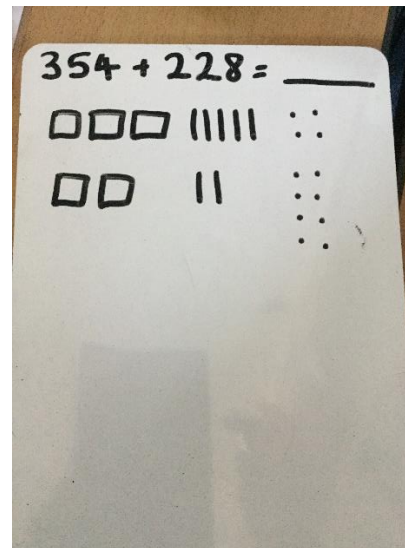
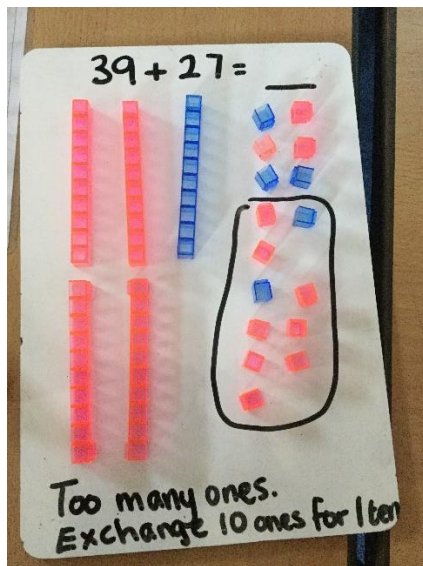
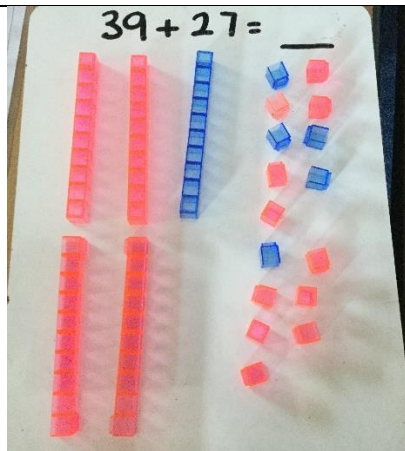
42 + 34 = 76

Add the tens

1. 324 + 143 = 467

324
+ 143
467

Compact
method- with
exchanging



$39 + 27 = \underline{\quad}$

Too many ones.
Exchange 10 ones for 1 ten

$39 + 27 = \underline{66}$

Too many ones.
Exchange 10 ones for 1 ten

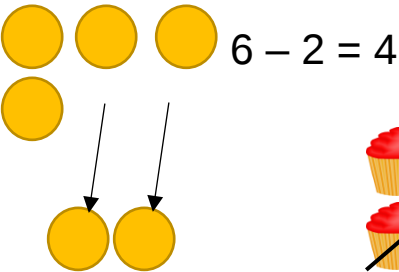
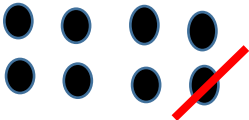
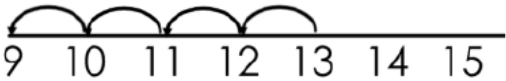
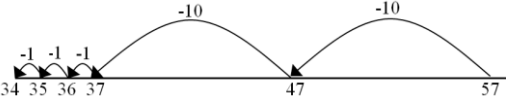
$354 + 228 = \underline{582}$

Too many ones.
Exchange 10 ones for one 10.

4. £ 33.59 + £ 7.55 =
£ 41.14

5. 23.361 + 9.08 =
32.441

Subtraction

Objective and Strategies	Concrete	Pictorial	Abstract
Taking away ones and counting back “What’s the difference between ...” and “... take away ..” to reinforce interchangeability	Use physical objects, counters, cubes etc to show how objects can be taken away and to reinforce one to one correspondence  $6 - 2 = 4$  Use counters and move them away from the group as you take them away counting backwards as you go. 	Cross out drawn objects to show what has been taken away.  $8 - 1 = 7$ Count back on a number line or number track  Start at the bigger number and count back the smaller number showing the jumps on the number line.  This can progress all the way to counting back using two 2 digit numbers.	$18 - 3 = 15$ $8 - 2 = 6$ Put the first number in your head, count back the second number. What number are you at? Use your fingers to help count back.

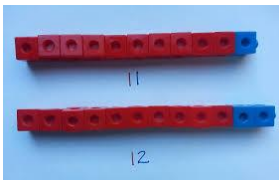
Find the difference

“What is the difference between ...?”

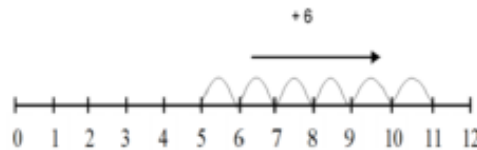
“How many more?”

“How many less?”

Compare amounts and objects to find the difference. Don't use number sentence yet – just the language what is the difference



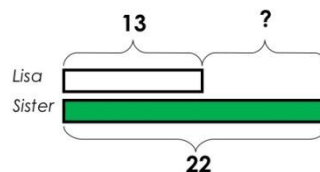
Use cubes to build towers or make bars to find the difference



Count on to find the difference.

Comparison Bar Models

Lisa is 13 years old. Her sister is 22 years old.
Find the difference in age between them.



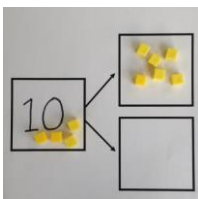
Draw bars to find the difference between 2 numbers.

Chn introduced to the **operation**.
We show find the difference as a **minus sign**.

Chn use this to solve difference between and more and fewer word problems.

Sophia has 8 cakes and Tom had 6 cakes.
How many more cakes does Sophia have?

Part Part Whole Model

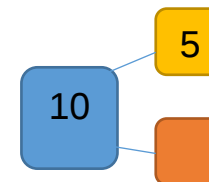
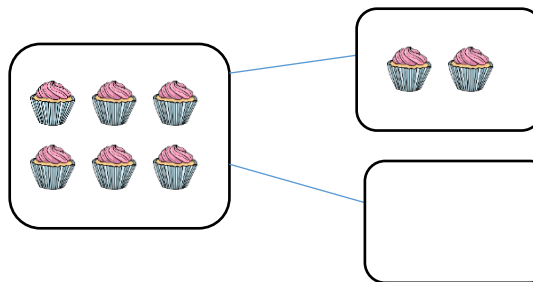


Link to addition- use the part whole model to help explain the inverse between addition and subtraction.

If 10 is the whole and 6 is one of the parts. What is the other part?

$$10 - 6 =$$

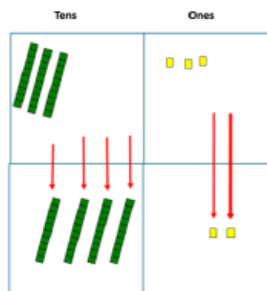
Use a pictorial representation of objects to show the part part whole model.



Move to using numbers within the part whole model.

Compact method without exchanging

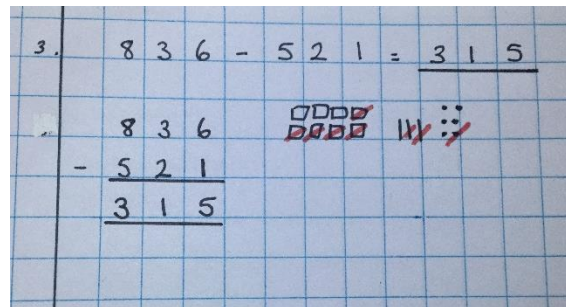
$$75 - 42 = 33$$



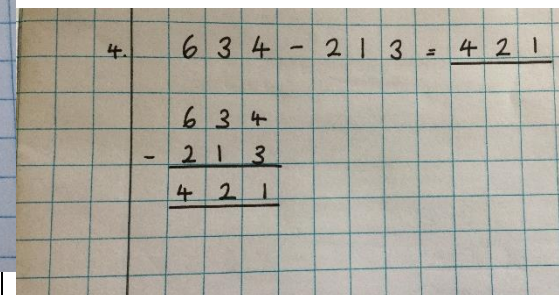
Use dienes to make the first number then take the second number away.

Start by taking away the ones first.

Draw the dienes alongside the written calculation to help to show working.



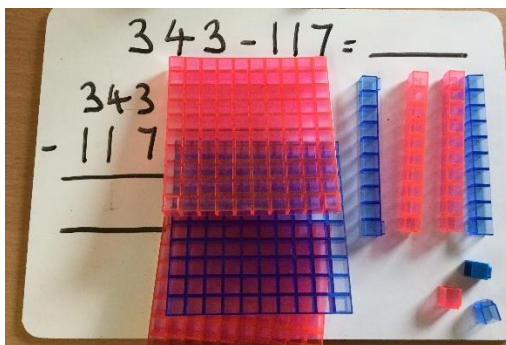
This will lead to a clear written column subtraction.



Compact method with exchanging

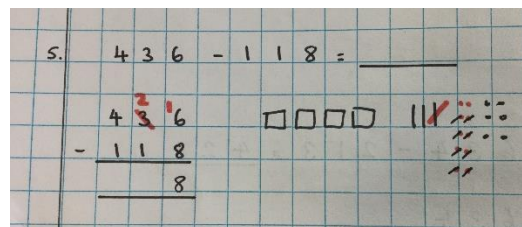
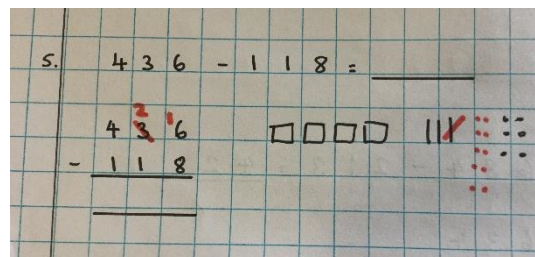
Using dienes, start with one exchange before moving onto subtractions with 2 exchanges.

Make the first number with dienes

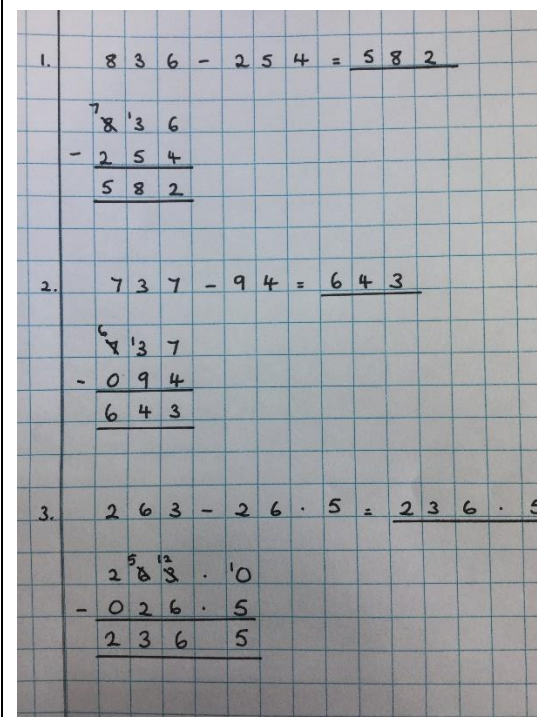


Start with the ones, do I have enough ones? I need to exchange one of my

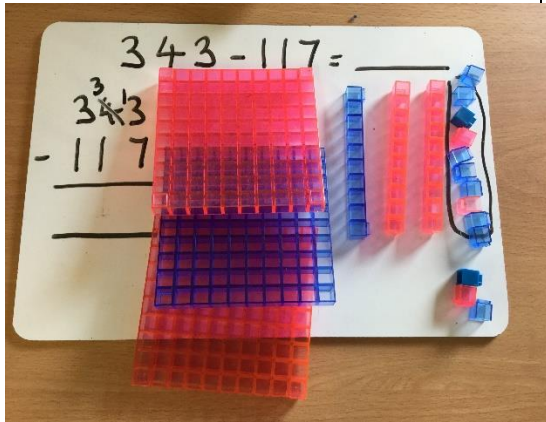
Draw the dienes and show what you have taken away by crossing the dienes out as well as clearly showing the exchanges you make.



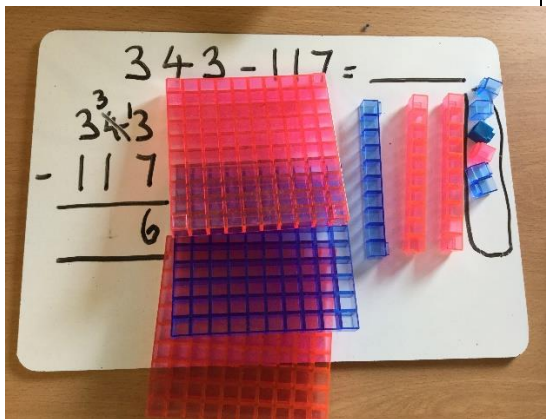
This will lead to an understanding of subtracting any number including decimals.



tens for ten ones.



Now I can subtract my ones.



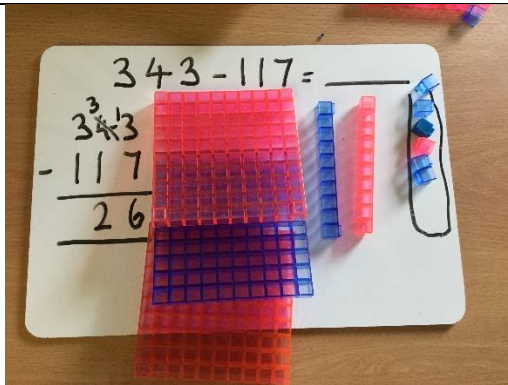
Now look at the tens.

$$\begin{array}{r} 436 \\ - 118 \\ \hline 18 \end{array}$$

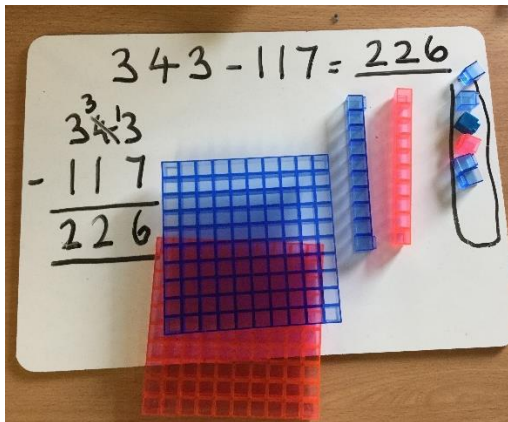
□□□□

$$\begin{array}{r} 436 \\ - 118 \\ \hline 318 \end{array}$$

□□□□



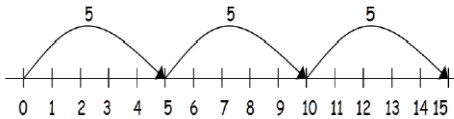
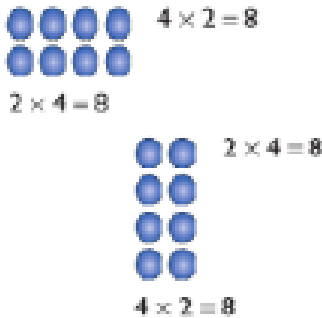
Then the hundreds.



Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.

Multiplication

Year Group expectations: End of Year 2 – fluent in 2, 5 and 10 tables up to 12x. (2 times 4 is 8 etc not skip counting)
 End of Year 3 – fluent in 2,3,4,5,8,10 tables up to 12x with associated division facts
 End of Year 4 – fluent in all 12 times tables up to 12 x with associated division facts

Objective and Strategies	Concrete	Pictorial	Abstract
Repeated addition	 $3 + 3 + 3 = \underline{9}$ $3 \times 3 = \underline{9}$	 $5 + 5 + 5 = \underline{15}$ $3 \times 5 = \underline{15}$ $4 + 4 = \underline{8}$ $2 \times 4 = \underline{8}$ 	$3 \times 6 = \square$ $6 + 6 + 6 = \square$ $3 + 3 + 3 + 3 + 3 + 3 = \square$
Arrays- showing commutative multiplication	Create arrays using counters/ cubes to show multiplication sentences.  $4 \times 6 = \underline{24}$ $6 \times 4 = \underline{24}$	Draw  arrays in different rotations to find commutative multiplication sentences.	Use an array to write multiplication sentences . If $5 \times 3 = 15$ Then $3 \times 5 = \square$

Compact Method

$$13 \times 4 = \square$$

Use dienes to show the partitioning of the 2-digit number.

x	T	U

4 lots of 13

$$40 + 12 = 52$$

Multiply the ones, multiply the tens. Add them back together

$$\begin{array}{r}
 3. \quad 27 \times 3 = \underline{81} \\
 \begin{array}{r}
 27 \\
 \times 3 \\
 \hline
 21 \quad 7 \times 3 \\
 60 \quad 20 \times 3 \\
 \hline
 81
 \end{array}
 \end{array}$$

$$10 \times 4 = 40$$

$$3 \times 4 = 12$$

$$\begin{array}{r}
 5. \quad 87 \times 4 = \underline{348} \\
 \begin{array}{r}
 87 \\
 \times 4 \\
 \hline
 028 \quad 7 \times 4 \\
 320 \quad 80 \times 4 \\
 \hline
 348
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 3. \quad 75 \times 6 = \underline{450} \\
 \begin{array}{r}
 75 \\
 \times 6 \\
 \hline
 450
 \end{array}
 \end{array}$$

$$7. \quad 32 \times 24 = \underline{768}$$

$$\begin{array}{r} 32 \\ \times 24 \\ \hline 128 \\ 640 \\ \hline 768 \end{array}$$

$$2. \quad 24 \times 68 = \underline{1632}$$

$$\begin{array}{r} 24 \\ \times 68 \\ \hline 0192 \\ 1440 \\ \hline 1632 \end{array}$$

$$1. \quad 22.4 \times 3 = \underline{67.2}$$

$$\begin{array}{r} 22.4 \\ \times \quad 3 \\ \hline 67.2 \end{array}$$

$$2. \quad 33.6 \times 2.3 = \underline{77.28}$$

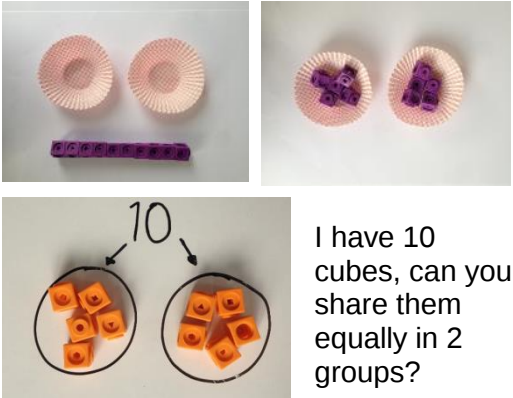
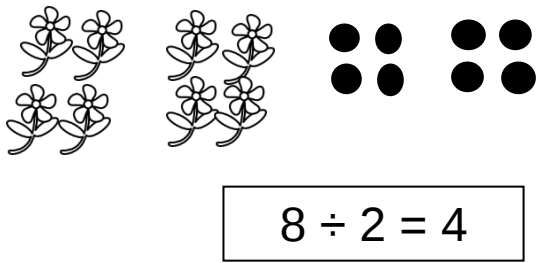
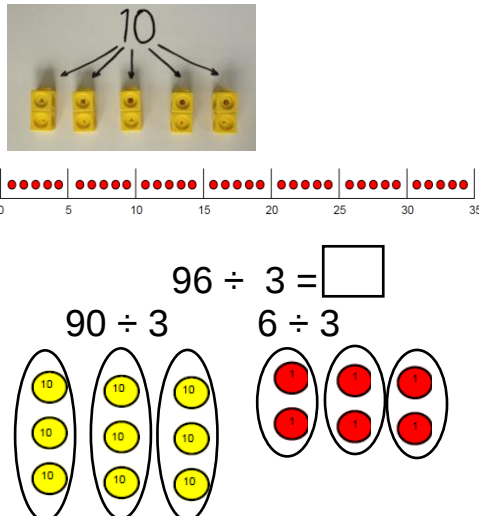
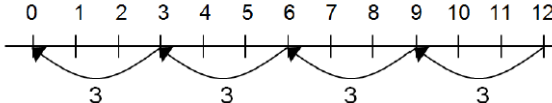
$$\begin{array}{r} 33.6 \\ \times 2.3 \\ \hline 1008 \\ 6720 \\ \hline 77.28 \end{array}$$

2 decimal places in question

The decimal point is put into the answer at the end of the calculation

2 decimal places in the answer

Division

Objective and Strategies	Concrete	Pictorial	Abstract
Sharing objects into groups	 I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or draw dienes to share quantities into equal groups. 	Share 9 buns between three people. $9 \div 3 = 3$
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  $96 \div 3 = \square$ $90 \div 3 \quad 6 \div 3$	Use a number line to show jumps in groups. The number of jumps equals the number of groups. $12 \div 3 = \underline{4}$ 	$28 \div 7 = 4$ Divide 28 into 7 groups. How many are in each group?

Division within arrays	Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg $15 \div 3 = 5$ $5 \times 3 = 15$ $15 \div 5 = 3$ $3 \times 5 = 15$	Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. $7 \times 4 = 28$ $4 \times 7 = 28$ $28 \div 7 = 4$ $28 \div 4 = 7$
Division with a remainder	$14 \div 3 =$ Divide objects between groups and see how much is left over	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder. Draw dots and group them to divide an amount and clearly show a remainder.	Complete written divisions and show the remainder using r. $29 \div 8 = 3 \text{ REMAINDER } 5$ ↑ dividend ↑ divisor ↑ quotient ↑ remainder

Compact division

Begin with divisions that divide equally with no remainder.

1. $872 \div 4 = \underline{218}$

$4 \overline{)872}$

Move onto divisions with a remainder.

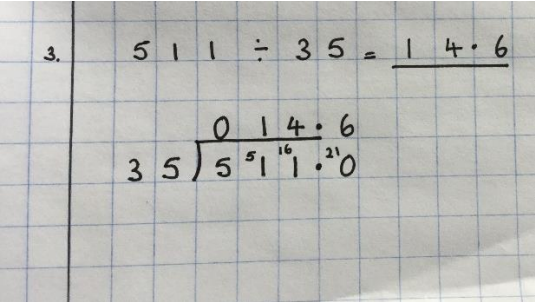
2. $432 \div 5 = \underline{86 \text{ r } 2}$

$5 \overline{)432} \text{ r } 2$

Show the remainder as a fraction. Use the language 86 and 2 out of the next group of 5.

$$432 \div 5 = 86 \frac{2}{5}$$

Finally move into decimal places to divide

			the total accurately.  Use repeated addition clouds to support related multiplication facts when dividing by a 2-digit number.
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